Linear Forecast Combinations under MSE Loss and Forecast Breakdowns for Exchange Rates

Abstract

Forecast combinations models currently provide evidence of producing better forecasts than single forecast models. In this paper we produce an optimized combination in which a loss function characterized by mean squared error is used. The evaluation of the performance of each forecast used in the combination is in order to justify the deviation from the equal weighting model and we employ the Diebold-Mariano statistic. The results indicate that survey data outperform combined forecast models for exchange rates using quarterly data. The evidence provided in this paper is important, particularly for policy makers.

1. Introduction

A general finding in the exchange rates forecast literature shows that the evidence is mixed regarding results that exceed the predictions associated with models based on random walk specifications. While there is some evidence in favour of the forecasting ability in-sample, the consensus across the academic and practitioners literature is not clear to whether these results are consistent and robust out-of-sample. This finding is in large based on the observations made by Meese and Rogoff (1983) in which economic and monetary aggregates are not capable of producing good forecasts of future changes in the spot rate. Rossi (2006) looked at this problem and shed light on the nature of the lack of forecast ability of models based on economic variables. She concludes that studying parameter instability can contribute to produce better forecasts than those generated by random walk predictions.

Another issue related to exchange rates forecasting are the statistical tests applied to the data used to make such projections. To date, the evidence suggests that the tests applied to the time-series of the exchange rates under study used to explore and understand the data can play an important role in order to produce good forecasts, especially now with recent findings suggesting that the nature of the true generating process of the data can be non-linear. For example, Sweeney (2006) detected that systematic movements of nominal rates for the G-10 countries towards stable long-run equilibriums are in fact non-linear. Thus given the unit root test applied looking at the stationary condition necessary to accept the mean reversion, attention needs to be paid to the generating data process. For example, Sarno (2001), looking at the behavior of US public debt, finds that the US debt-GDP ratio can be described with a nonlinearly mean-reverting stochastic process and proposes the typical ADF unit root test cannot pick the non-linearity of the process.

In addition to the tests applied to the data looking for assistance in producing good forecasting models it is also common knowledge in the literature that some other considerations need to be taken into account to make forecasts. For example, one important issue is the evidence in current research showing that the short-term dynamic of the exchange rate follows a I(1) process and the long-term exchange rate equilibrium can be described reasonably well by I(0) process. The implications of specifying I(0) or I(1) processes lead the observer to different model specifications; thus while the I(1) forecasts look to predict the short-term dynamic of the exchange rate the I(0) assumption aims to study the projection from the mean reversion perspective.

This paper focuses on forecast combinations and forecast breakdowns. The contribution is related to forecasting exchange rates based on quarterly data using a combined forecasts in which time-series of historical data and survey data are employ in the combined model. The motivation relates forecast breakdowns to the performance of the combined model in which the Diebold-Mariano tests is used to evaluate. The justification of this procedure is based on the argument provided by Giacomini and Rossi (2009) where they note that the performance of the model in the future can be track down to evaluate consistency.

1.1. Survey Data

To date there is substantial literature on survey data. The general discussion focuses on the importance of this area for the expectations formation process. Pesaran and Weale (2006) provided a large summary of definitions, applications and methods used across current literature dealing with survey expectations. They find in survey data valuable information which can enhance the performance of forecasts. One early contribution looking at exchange rate forecast found in Fankel and Froot (1987), using survey data to forecast exchange rates, suggests that this information is superior to the forwards rate to produce forecasts. Fankel and Chinn (1993) exploit survey data to forecast exchange rates from the G-7 to the US dollar and report evidence in favour of this information to produce forecasts.

We begin part one by introducing some stylized facts reported in the literature related to exchange rates forecasting and in addition the inclusion of survey data into the forecast is justified. The rest of the paper is organized as follows: Section 2, addresses forecast combinations and forecast breakdowns. Section 3, develops forecast breakdowns procedure. Section 4, sets out the data and the model specification. Section 5, reports the results of the application. Section 6, concludes.

2. Forecast Combinations and Forecast Breakdowns

Forecast combinations have been the subject of considerable attention in recent work. Since Clement (1989) this area of research has gained general acceptance based on encouraging results reported in the literature, such as Pesaran and Pick (2010) and Elliott, G., C.W.J. Granger and A. Timmermann (2006). Both of these papers report that combinations of different forecasts typically result in more accurate projections than single forecasts. While Elliott, G., C.W.J. Granger and A. Timmermann (2006) suggests that equal weighting seems natural for the mix of the combination he also finds it interesting to explore the idea of working with optimal weight combinations for the following case, $\lambda 1 \neq \lambda 2$ where λ stands for a particular weight in the mixing. His argument for equal weighting is based on the idea that individual forecasts, whose parameters are estimated recursively, could be affected by bias estimates if the combination of the weights differs from the following case $\lambda 1 = \lambda 2 = 0,5$. Pesaran and Pick (2010) study forecast combinations and look at averaging forecasts over different estimation windows. They find this approach capable of generating forecasts reasonably robust to structural breaks.

In the econometric literature a structural break is defined as a change in the parameter of the system. Hendry and Mizon (1998) indicate that the structural break occurs when the parameters of a conditional distribution are not time-constants. The importance of this issue relates to the properties of the framework proposed by Giacomini and Rossi (2009) where they show that forecast breakdowns can be caused by instabilities in the data generating process; this issue relates to the properties of their forecast breakdown test which is used in this paper. The implication of the presence of breaks is also discussed in Elliott, G., C.W.J. Granger and A. Timmermann (2006). He suggests that this issue needs to be taken into account, arguing that the instability in the data generating process can affect the estimation of the weights as it may cause underperformance relative to that of the best individual forecasting.

2.1. Detecting Forecast Breakdowns

Giacomini and Rossi (2009) introduced forecasts breakdowns as a formalization of the deterioration of the forecasting performance in the out-of-sample forecast. Their suggestion is very much along the lines of a large literature looking at forecasting exchange rates in which the performance of the forecast in-sample cannot be reproduced out-of-sample. They propose a recursive scheme, which is adopted in this paper, to compare forecasts in-sample with forecasts out-of-sample where they assess the surprise loss defined as the difference between the out-of-sample loss with the average in-sample loss. In this paper we explore this idea but we consider the Diebold-Mariano tests to compare the forecasts.

In general the Diebold-Mariano test follows that $\{yt\}$ denotes the series to be forecast and $y^1_{t+h|t}$ and $y^2_{t+h|t}$ are two competing forecasts of y_{t+h} based on Ω_t. In this paper $y^1_{t+h|t}$ is computed from a AR(1) model and $y^2_{t+h|t}$ are survey data forecasts obtained from OECD databank . The forecast errors from the two competing models are described in equations (1) and (2),

$$\varepsilon^1_{t+h|t} = y_{t+h} - y^1_{t+h|t} \tag{1}$$

$$\varepsilon^2_{t+h|t} = y_{t+h} - y^2_{t+h|t} \tag{2}$$

The h-steps forecasts are computed from t,T with $t = t_0, ..., T$ and the series of errors are shown bellow,

$$\{\varepsilon^1_{t+h|t}\}^T_{t_0}, \{\varepsilon^2_{t+h|t}\}^T_{t_0},$$

The accuracy of the forecasts is measured using the square error loss function thus the resulting loss function has the following form,

$$\mathrm{L}(\varepsilon^i_{t+h|t}) = (\varepsilon^i_{t+h|t})^2, i = 1, 2$$

In order to find which model produces more accurate forecasts the null and the alternative hypothesis are respectively,

$$H_0 : \mathrm{E}[L(\varepsilon^1_{t+h|t})] = \mathrm{E}[L(\varepsilon^2_{t+h|t})]$$

and,

$$H_1 : \mathrm{E}[L(\varepsilon^1_{t+h|t})] \neq \mathrm{E}[L(\varepsilon^2_{t+h|t})]$$

With the null of equal predictive accuracy given by

$$H_0 : \mathrm{E}[d_t] = 0$$

The Diebold-Mariano test follows that under the null hypothesis the Diebold-Mariano statistic has an asymptotic standard normal distribution. This is important because the long run variance is used by the statistic computed in the test. Harvey, Stephen and Newbold (1997) notes that this test, based on the loss differential, is likely to produce good comparison of competing forecasts thus the loss differential is given by

$$d_t = L\left(\varepsilon^{h,1}_t\right) - L\left(\varepsilon^{h,2}_t\right)$$

The Diebold-Mariano test may be written as equation (3) where the null of equal predictive accuracy is rejected with a significance level of 5% if Λ_τ >1.96

$$\Lambda_\tau = \varsigma^{-1}_\tau \mu_\tau \tag{3}$$

with,

$$\varsigma_\tau^2 = 2\sum_{j=0}^{\tau} \operatorname{cov}(d_t, d_{t-j})$$

and,

$$\mu_\tau = \tau^{-1}\sum_{t=1}^{\tau} d_t$$

$$\Lambda_\tau \sim \mathrm{N}(0,1)$$

3. Data and Model Specification

Our data set comprises two sets of information for each exchange rate under study. Specifically, the information sets are survey data forecasts obtained from OECD databank and the observed exchange rate. All currencies are denominated in US dollars and the countries considered to produce such forecasts are Australia (AUS), Canada (CAN), Czech Republic (CZE), the Euro zone (EUR), Japan (JPY), United Kingdom (GBP), Sweden (SWE), Norway (NOR) and Switzerland (SWI). The frequency of the data sample is quarterly data and the analysis is implemented from Q1-1970 to Q1-2010. Two special cases are the data for the Check Republic and the Euro zone in which the data sample is from Q3-1994 and Q4-1998 respectively.

This paper considers two different models in which individual forecasts of the exchange rates specified above are used in combination to produce a combined forecast. Thus this study relates changes in the currency rate to past values of the same exchange rate computed using an AR(1) model and to changes using survey data forecasts.

The general forecasting model is described in equations (4) and (5),

$$s_{t+h} - s_t = \lambda_{1,t}\left(g_t^h - s_t\right) + \lambda_{2,t}\left(f_t^h - s_t\right) \tag{4}$$

$$\Delta^h s_t = \lambda_{1,t}\left(g_t^h - s_t\right) + \lambda_{2,t}\left(f_t^h - s_t\right) \tag{5}$$

$$s_t = \log S_t$$

$$\Delta^h s_t = s_{t+h} - s_t, h \in \frac{3}{12}$$

$$f_t^h = \log F_{t+h}$$

$$g_t^h = \mathbf{E}_t(logS_{t+h}|\Omega(S_t))$$

where s_{t+h} is the one step ahead forecast of the exchange rate under consideration, s_t is the log of the spot price in "t" of the exchange rate, $\lambda_{1,t}$ and $\lambda_{2,t}$ are weights used to make the forecast combination thus respectively they are associated with the forecasts produced with observed exchange rates and with the survey data forecasts. f_t^h is the log of the survey data and g_t^h is a forecast produced using and AR(1) process.

While the combination in *Model 1* is based on equal weighting, *Model 2* uses a combination in which weights are time variant. The idea of using time varying weights is to allow an assessment of the impact of an optimized combined forecast.

Thus the specification on the weights follows that,

Moldel 1:

$$\lambda_{1,t} = \lambda_{2,t} = 0.5$$

Model 2:

$$\lambda_{1,t} + \lambda_{2,t} = 1, \quad \lambda_{1,t} > 0$$

The model is set up using one constraint which is the weights have to sum to unity. This restriction drew attention to the idea that essentially all the information available can be captured by the two weights. According to Timmerman (2006) imposing this constraint can lead to efficiency gains.

4. Analysis of the Forecasts

Plots of the weights used to produce the forecast combinations of the survey data with the forecasts from the simple time-series regression for each of the exchange rates under analysis are shown in Figure 1. Additionally, in this figure the extent of change in the evolution of the weights over time

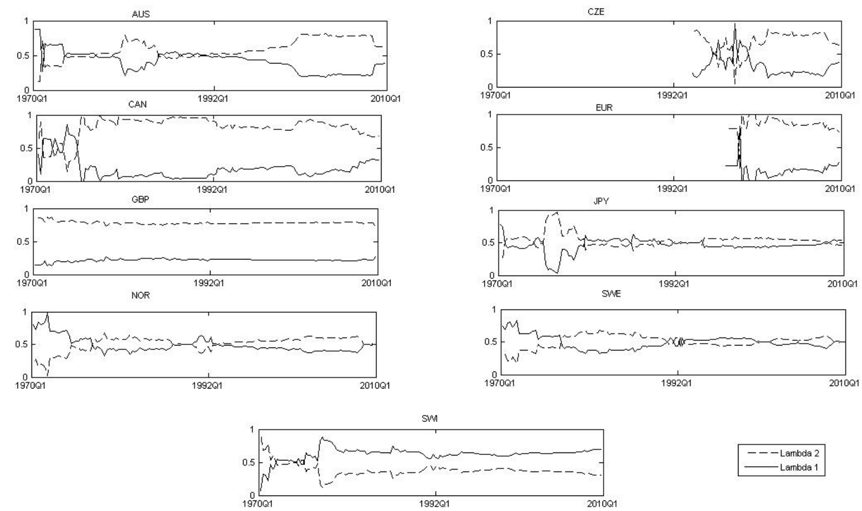

Figure 1: Optimal weights used in the forecast combination.Australia (AUS), Canada (CAN), Check Republic (CZE), the Euro zone (EUR), Japan (JPY), United Kingdom (GBP), Sweden (SWE), Norway (NOR) and Switzerland (SWI).

can also be observed. Elliot and Timmerman (2008) suggest that weights change over time due to their ability to adapt to structural breaks. Figure 1 shows for all exchange rates, with the exception of AUS and CZE a pattern in which the weights seem stable over time particularly after the economic crisis of the 1980's. It is clear from analysing this figure that the weights experiment changes during periods of financial distress. For example, during the Asian crisis in 1998 the weights shifted from their average, and they also shifted during the past financial crisis of 2008. This shift is picked up by the Diebold-Mariano test plot in Figure 2. Another pattern that can be observed in Figure 1 is that after a financial crisis the weighting turns to the average.

Table 1 contains the mean and the standard deviation of the optimized weight associated to the forecast produced by the simple time-series regression for three different time samples. The table reports for the CAN, EUR and GBP exchange rates that the optimal weight used in the combination

Table 1: Weight ’$\lambda 1$’ used to produce forecasts combinations

Currency / $\lambda 1$	AUS	CAN	CZE	EUR	GBP	JPY	NOR	SWE	SWI
Mean (1)	0.41	0.19	0.25	0.19	0.22	0.54	0.49	0.49	0.38
Std. Dev.	0.15	0.17	0.20	0.11	0.02	0.10	0.11	0.10	0.10
Mean (2)	0.41	0.20	0.49	0.22	0.22	0.56	0.45	0.55	0.38
Std. Dev.	0.03	0.01	0.21	0.00	0.00	0.02	0.01	0.01	0.01
Mean (3)	0.26	0.26	0.23	0.17	0.22	0.50	0.42	0.45	0.33
Std. Dev.	0.07	0.06	0.07	0.04	0.01	0.03	0.05	0.04	0.02

Table 1 reports the mean and standard deviation for $\lambda 1$ which is an optimized weight, worked out recursively, used in the combination of the survey data with the forecasts from the simple time-series regression. Given the specifications of the model described by equations (4) and (5) in which $\lambda 2 = 1 - \lambda 1$ we report the data only for this weight. Mean (1) is the average $\lambda 1$ computed using all the data, Mean (2) is the average $\lambda 1$ computed using the Asian crisis data, and Mean (3) is the average $\lambda 1$ computed using the Subprime crisis data.

is stable at about 25%, this indicates that the forecasts taken from the survey data are relatively more important than the forecasts from the simple time-series regression, also they are consistent over time. The exchange rates NOR, and SWE have a forecast combination which stabilizes the weight around 42% and 45% after the subprime crisis. Note that the currencies JPY, NOR, and SWE have a weight stabilizing around 50% when the full data sample is used to compute the optimized weight.This indicates that these last three exchange rates have an optimal weight very close to the equal weight condition suggested in the literature.

Table 2 reports mean square errors for all exchange rates individually. The table shows data for the full sample and in addition reports data for two sub-samples related to the Asian crisis and to the Subprime crisis. For, AUS, CAN, CZE, NOR, SWE, SWI the data reported in the first column, for all three samples, contains the smaller mean square errors. For the case of the EUR the smaller square error is found at the Asian crisis sample for the case of the optimized combination. The exchange rate GBP show the smaller square error in the full sample, and in the Subprime crisis forecast combination. Finally JPY reports that the best case is for the sample Subprime crisis forecast combination. This table shows that survey data represents a real alternative to forecasts combination if quarterly data is employ to produce mix models. Nevertheless, for the cases of GBP and JPY is reveal that

Table 2: Recursive Mean Square Errors

		MSE Results			
		Survey	Combination	AR	50-50
AUS	All Sample	0.018	0.073	0.046	0.032
	Asian Crisis	0.021	0.044	0.041	0.031
	Subprime Crisis	0.069	0.200	0.109	0.089
CAN	All Sample	0.007	0.021	0.014	0.011
	Asian Crisis	0.004	0.010	0.006	0.005
	Subprime Crisis	0.038	0.098	0.063	0.050
CZE	All Sample	0.023	0.050	0.034	0.029
	Asian Crisis	0.036	0.108	0.087	0.062
	Subprime Crisis	0.055	0.120	0.103	0.079
EUR	All Sample	0.014	0.037	0.030	0.022
	Asian Crisis	0.033	0.016	0.018	0.025
	Subprime Crisis	0.023	0.056	0.043	0.033
GBP	All Sample	0.027	0.024	0.045	0.036
	Asian Crisis	0.008	0.016	0.010	0.009
	Subprime Crisis	0.036	0.033	0.097	0.067
JPY	All Sample	0.018	0.105	0.061	0.040
	Asian Crisis	0.041	0.226	0.111	0.076
	Subprime Crisis	0.036	0.220	0.100	0.068
NOR	All Sample	0.015	0.079	0.047	0.031
	Asian Crisis	0.014	0.049	0.031	0.022
	Subprime Crisis	0.044	0.185	0.115	0.080
SWE	All Sample	0.020	0.084	0.053	0.037
	Asian Crisis	0.014	0.033	0.022	0.018
	Subprime Crisis	0.038	0.108	0.082	0.060
SWI	All Sample	0.023	0.117	0.069	0.046
	Asian Crisis	0.015	0.069	0.042	0.028
	Subprime Crisis	0.029	0.090	0.067	0.048

Table 2 reports the results of the mean square error. This measure corresponds to the quadratic loss function computed by a recursive system used to estimate the optimal weights employed to produce the forecast combination. The errors reported in the first column denoted the data for the Survey Data forecast, the second column relates the errors to the Forecast Combination, The third column report the time-series regression computed as an AR(1) process and finally the fourth column indicates the errors of a forecast combination in which the forecast combination uses equal weights thus 50-50.

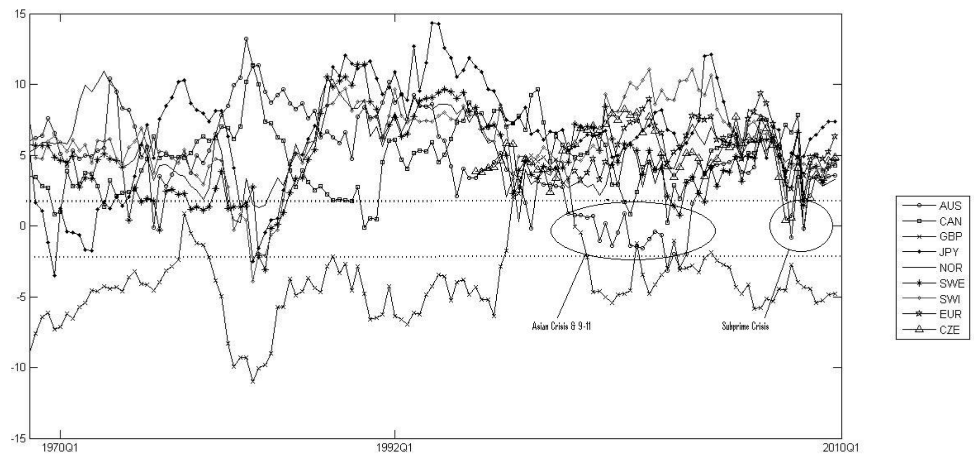

Figure 2: Diebold-Mariano statistic. In the circles is indicated the Asian Crisis and the Subprime Crisis.

optimal models using combined forecast outperform survey data.

Figure 2 show the Diebold-Mariano statistic used to assist in this evaluation of the forecasts performance. This test captures the forecasts breakdowns for the sample periods discussed above thus in both the Asian and Subprime crisis the Diebold-Mariano statistic changes, accepting the equal weighting hypothesis. This is confirmed by Table 1 in which the change in the weights for this sample period allows the combination to vary adapting the model to incorporate new information that relates financial distress.

5. Summary

In this paper forecast combinations and forecast breakdowns are study using a sample of important exchange rates follow to date in financial markets. To produce the forecast combinations a mix of time-series of the exchange rates in the form of an AR(1) system with survey data forecasts is used.

We report mix evidence using quarterly data in favor of optimized forecasts combination based on mean square errors loss function. In addition, we report survey data being important in the expectation formation process and we find evidence in favor of using this type of information. The implications of the forecast breakdowns procedure for market participants and specially for policy makers based on quarterly data is important as the data shows evidence of this approach being capable to adapt well to new information.

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