

THE IMPACT OF TRANSACTION COSTS IN PORTFOLIO OPTIMIZATION: A COMPARATIVE ANALYSIS BETWEEN THE COST OF TRADING IN PERU AND THE UNITED STATES

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Abstract

In general, investors try to hold the most efficient portfolio of stocks, which is done by rebalancing the portfolio from time to time by either buying or selling stocks. However, rebalancing a portfolio incurs transaction costs that often hamper the performance of the portfolio, resulting in less revenue or return than initially expected. Over a few dimensions as type and number of stocks, holding period, and trading strategy, the transaction costs structure applied in Peru has been compared and analyzed with respect to the ones applied in the U.S. We conclude that a retail investor actively trading in Peru can pay up to fourteen times more in transaction costs than trading the same portfolio but in the U.S. These comparatively high transactions costs prevent retail investors to trade in the Peruvian stock market while fueling illiquidity to this market.

JEL Classification: G11, G12

Keywords: Portfolio optimization, transaction costs, portfolio turnover

1 Introduction

The Peruvian stock exchange, known as the *Bolsa de Valores de Lima* (BVL), currently lists 278 securities with a total market capitalization of about 130 billion USD.¹ It is regulated by the *Superintendencia del Mercado de Valores* (SMV) and has a limited exchange self-regulatory organization. The BVL is currently facing serious liquidity and low transaction volume problems² and its average daily trading volume in 2016 was 18.26 million USD. On the other hand, the New York Stock Exchange (NYSE) is the biggest stock market in the world with a market capitalization of nearly 20.6 trillion USD.³ It is regulated by the Securities and Exchange Commission (SEC). The NYSE is characterized by its high liquidity and its large stock inflows and outflows that incur on a daily basis: around 42 billion USD are traded every day and has more than 2700 listed firms. The Sustainable Stock Exchange Initiative considers the NYSE as one of the most advanced stock exchanges in the world. Because of its quality standards, the NYSE has been selected as a benchmark to compare transaction costs with the ones of the BVL.

We define transaction costs (TC) as the cost of buying or selling securities in order to rebalance or build a particular portfolio. The TC structure in Peru is interesting in the sense that it charges investors a percentage of the stock value traded. However, there is also a minimum trading fee to pay that is high and finally makes the Peruvian stock market very expensive with respect to the ones of more developed markets. In the U.S. there exists two types of TC. The first one charges investors per the number of shares they buy or sell and it is called TC per share. The second structure charges investors per the number of trades they make and it is called TC per

¹ Monthly report of the BVL. April 2017.

² In 2016, MSCI was close to consider Peru as a frontier market instead of an emerging one.

³ Data taken from the NYSE Group Shares Outstanding and Market Capitalization of Companies Listed (March 2017): nyxdata.com.

trade. The comparison of the aforementioned TC structures with the one applied in Peru (TC per percentage) is used in order to determine which type of TC is the most appropriate and under which conditions this said type could happen.

To answer the previous research question, we analyze the behavior of the TC structures in Peru and the U.S. over three dimensions: the type and number of stocks, the holding period of the portfolio, and the selected trading strategy. The paper concludes that TC per share is the cheapest when small monetary amounts are invested in the portfolio. But once a certain initial amount is reached, TC per trade become preferable. However, the TC structure applied in Peru represents the most expensive structure for any dimension. This analysis was carried on using current cost parameters found in both the BVL and the NYSE. To the best of our knowledge, there has not been any study analyzing the cost of trading in Peru and assessing the convenience of the TC structure applied on the BVL. Also, as a by-product of our analysis, we corroborate some empirical regularities like that TC are higher for portfolios composed of small market capitalizations stocks, holding period helps dilute entry and exit costs and active trading strategies are subject to higher TC.⁴

In conclusion, retail investors that decide to invest in the Peruvian stock market can only do so if they adopt a buy and hold investment strategy. In fact, they would be facing substantial transaction costs by rebalancing their portfolio from month to month. This is more dramatic for small initial investment amounts. For example, if 30K USD are invested in Peruvian stocks (under TC per percentage), the investor can lose up to 10% in yearly return due to TC. In

⁴ Relevant literature regarding optimal strategies in the presence of TC can be found in Davis and Norman (1990) and Leland (1999). The effects of TC in asset pricing is treated in Chalmers and Kladek (1998) and the references therein. Cornuejols and Tütüncü (2007) treat Mean-Variance portfolio optimization problems in the presence of transaction costs providing useful transformations to make the aforementioned problems solvable using traditional linear and quadratic programming techniques. Gaivoronski et al. (2005) and Birge and Chavez-Bedoya (2013) implements index tracking and "passive" strategies in the presence of TC.

comparison, trading the same stocks in the U.S. under the other TC structures incurs 1% in yearly return. The main issue is that the minimum trading fee applied in Peru is very expensive compared to the stock inflows and outflows that are actually incurred. Therefore, portfolio rebalancing and active portfolio management strategies could be adopted in the BVL only if the minimum trading fee was close to the one applied for TC per share in the U.S. To make the Peruvian TC per percentage competitive, a dramatic reduction in the minimum cost per trade is needed: it has to go from \$25 to \$1 to make the current cost structure competitive for a retail investor with a relatively small initial investment amount. This reduction in fees will lure a bigger number of investors to the BVL and it will increase its liquidity and volume negotiated.

This paper is divided into five different parts. Section 2 presents the methodology, the definitions of TC and the indicators used to assess portfolio performance. Section 3 contains the analysis and provides a deep understanding of the behavior TC under the different dimensions. Section 4 aims at making TC per percentage competitive to TC per share and TC per trade by changing its cost parameters. Finally, Section 5 concludes and provides recommendations from the perspective of an investor trading in the Peruvian stock market.

2 Methodology

This section defines the different trading strategies and the three types of TC used in this paper. It also shows how we measure the impact of TC in portfolio performance and how it is calculated.

2.1. Trading strategies

Three trading strategies are going to be studied in this paper. The objective for each strategy is to converge to specific portfolio weights at the end of every period. These target values represent the optimal allocation of a stock in the portfolio. The portfolio has to be rebalanced since the optimal weights are not achieved automatically due to price fluctuations. Let i denote a particular stock of a portfolio P of N stocks, and $x_{i,t+1}$ be the weight of stock i at the beginning of period $t + 1$. While including entry and exit costs, t oscillates from $t = 0$ to $t = T$, where 0 represents the initial period when the portfolio is built, and T represents the very last period when all stocks are sold and the portfolio liquidated. Therefore, t represents a specific period and oscillate from 0 to $T - 1$, where $T - 1$ is the number of periods.

The first trading strategy is the Equally Weighted (EW). This strategy sets the same weight allocation for every stock, at every period.⁵ This means that the weight for one stock has to remain constant and equals to the following:

$$x_{i,t+1} = \frac{1}{N}. \quad (1)$$

The second trading strategy is the Market Capitalization (MC). This strategy allocates a weight for each stock depending on its market capitalization within the portfolio.⁶ Let $MCap_{i,t}$ denote

⁵ The performance of this strategy was studied empirically in DeMiguel et al. (2007) concluding that is extremely robust and beats more sophisticated strategies in terms of the out-of-sample Sharpe Ratio.

⁶ This trading strategy is inspired in the work of Sharpe (1964) because the Capital Asset Pricing Model (CAPM) states that the optimal portfolio is a combination of the market portfolio and the risk-free rate. The market portfolio has weights proportional to the corresponding market capitalizations of the assets involved.

the market capitalization of a particular stock i at time t . The weight of a stock i at period $t + 1$ can be expressed as follows:

$$x_{i,t+1} = \frac{MCap_{i,t+1}}{\sum_{i=1}^N MCap_{i,t+1}}. \quad (2)$$

The third trading strategy is the Markowitz strategy (MZ) which is inspired in Markowitz (1952) and Sharpe (1964). This strategy aims at having the most efficient portfolio allocation in terms of a risk-return relationship. Weights are computed by maximizing the Sharpe Ratio. For each period, a covariance matrix and a mean vector based on the stock's previous returns have been computed. Let $SR_{P,t+1}$ denote the Sharpe Ratio of a portfolio P at period $t + 1$, $R_{P,t+1}$ the return of portfolio P at period $t + 1$, R_f the risk-free rate, and $\sigma_{P,t+1}$ the portfolio standard deviation at period $t + 1$. Weights $x_{i,t+1}$ for the N stocks of portfolio P are computed maximizing the following:

$$SR_{P,t+1} = \frac{R_{P,t+1} - R_f}{\sigma_{P,t+1}}. \quad (3)$$

In this paper we do not consider short sales, then the $x_{i,t+1}$ maximizing (3) will be forced to be greater than zero.

2.2. Unbalanced portfolio weights

Due to the change in stock prices at the end of every period, the weights of the stocks of the portfolio have to be rebalanced in order to be consistent with the trading strategies described in Section 2.1.

Let i denote a particular stock of a portfolio of N stocks, $x_{i,t}$ the target weight of a stock i at the beginning of period t , and $x_{i,t+1}^u$ the unbalanced weight of a stock i at the beginning of period $t + 1$. Let $r_{i,t+1}$ denote the return of a particular stock i at period $t + 1$. For $t \geq 0$ to $t = T - 1$, the unbalanced weight of a stock i at the beginning of period $t + 1$ can be expressed as follows:

$$x_{i,t+1}^u = x_{i,t} \frac{1+r_{i,t+1}}{1 + \sum_{i=1}^N x_{i,t} \times r_{i,t+1}}. \quad (4)$$

In order to describe the TC involved, it is assumed that $x_{i,t+1}^u \neq x_{i,t+1}$ for every t and i .

2.3. Transaction costs

The paper identifies three different types of TC: TC per share, TC per percentage, and TC per trade. Under TC per share, investors are charged a fixed fee for each single stock they buy or sell. TC per percentage charge investors a percentage of the stock value traded. TC per trade charge investors a specific amount for each trade they make. Next we describe each of the aforementioned TC structures in detail.

Let $P_{i,t}$ denote the price of stock i at the beginning of period t , and $W_{i,t}$ denotes the value of stock i in the portfolio at the beginning of period t . Let TC^s , TC^p and TC^e be TC per share, TC per percentage, and TC per trade, respectively. For $t = 0$ to $t = T - 1$, the TC per share, TC^s , can be expressed as follows:

if

$$x_{i,t+1} - x_{i,t+1}^u < 0, \quad (5)$$

then

$$TC_{i,t+1}^S = \text{Max} \left\{ \frac{|x_{i,t+1} - x_{i,t+1}^u| W_{i,t+1}}{P_{i,t+1}} \times \theta_i, \alpha_i \right\} + |x_{i,t+1} - x_{i,t+1}^u| W_{i,t+1} \times \rho_i, \quad (6)$$

else

$$TC_{i,t+1}^S = \text{Max} \left\{ \frac{|x_{i,t+1} - x_{i,t+1}^u| W_{i,t+1}}{P_{i,t+1}} \times \theta_i, \alpha_i \right\}, \quad (7)$$

where θ_i is the per-share trading fee in USD, α_i the minimum trading cost in USD, and ρ_i is the Regulatory Transaction Fee charged by the Securities and Exchange Commission (SEC) on stock sales only and when the trading occurs on the American stock market. The TC per share is a structure for TC commonly used in the U.S.

For $t = 0$ to $t = T - 1$, the TC per percentage, TC^P , can be expressed as follows:

$$TC_{i,t+1}^P = \text{Max} \{ |x_{i,t+1} - x_{i,t+1}^u| W_{i,t+1} \times \beta_i, \gamma_i \} + |x_{i,t+1} - x_{i,t+1}^u| W_{i,t+1} \times \mu_i, \quad (8)$$

where β_i represents the fixed percentage per stock value, γ_i the minimum trading cost in USD, and μ_i is the regulatory fee applied in Peru. The TC per percentage is the current Peruvian structure for TC.

Finally, for $t = 0$ to $t = T - 1$, the TC per trade, TC^e , can be expressed as follows:

if

$$x_{i,t+1} - x_{i,t+1}^u < 0, \quad (9)$$

then

$$TC_{i,t+1}^e = |x_{i,t+1} - x_{i,t+1}^u| W_{i,t+1} \times \rho_i + \varepsilon_i, \quad (10)$$

else

$$TC_{i,t+1}^e = \varepsilon_i , \quad (11)$$

where ε_i represents the trading cost in USD and ρ_i is the Regulatory Transaction Fee charged by the SEC on stock sales only. As in the case of the TC per share, the TC per trade is also an American structure for TC. The values of the coefficients θ_i , α_i , ρ_i , β_i , μ_i , and ε_i used for the numerical experiments will be defined in Section 3.

2.4. Entry and exit costs

Two key periods of the experiments are the very first one and the very last one. The very first period, referred as period 0, is the period during which the portfolio is fully built with the initial investment. It corresponds to a period of heavy stock purchases. This period is subject to a substantial amount of TC corresponding to large entry costs. Let In^j denote the entry costs associated with a particular type of TC j , either per share, per percentage or per trade. For $t = 0$ and for each type j of TC, entry costs, In^j , can be expressed as follows:

$$In^{TC^S} = \sum_{i=1}^N \text{Max} \left\{ \frac{x_{i,0} \times W_{i,0}}{P_{i,0}} \times \theta_i , \alpha_i \right\} , \quad (12)$$

$$In^{TC^P} = \sum_{i=1}^N \text{Max} \{ x_{i,0} \times W_{i,t=0} \times \beta_i , \gamma_i \} + x_{i,0} \times W_{i,t=0} \times \mu_i , \quad (13)$$

$$In^{TC^e} = \sum_{i=1}^N \varepsilon_i . \quad (14)$$

Similarly, the very last period, referred as period T corresponds to the period of liquidation of the portfolio and corresponds to a period of heavy stock sales and therefore subject to significant

TC. Let Out^j denote the exit costs associated with a particular type of TC j . For $t = T$, exit costs, Out^j , can be expressed as follows:

$$Out^{TC^s} = \sum_{i=1}^N Max \left\{ \frac{|x_{i,T} - x_{i,T}^u| W_{i,T}}{P_{i,T}} \times \theta_i, \alpha_i \right\} + |x_{i,T} - x_{i,T}^u| W_{i,T} \times \rho_i, \quad (15)$$

$$Out^{TC^p} = \sum_{i=1}^N Max \{ |x_{i,T} - x_{i,T}^u| W_{i,T} \times \beta_i, \gamma_i \} + |x_{i,T} - x_{i,T}^u| W_{i,T} \times \mu_i, \quad (16)$$

$$Out^{TC^e} = \sum_{i=1}^N |x_{i,T} - x_{i,T}^u| W_{i,T} \times \rho_i + \varepsilon_i. \quad (17)$$

With these assumptions, experiments were made with and without taking into consideration the entry and exit costs described in this section.

2.5. Turnover

Another variable that is analyzed in the paper is the turnover. It consists in the change in weight of a stock i after rebalancing. For $t \geq 0$ to $t = T - 1$, the turnover $Tur_{i,t+1}$ of a stock i at period $t + 1$ can be expressed as follows:

$$Tur_{i,t+1} = |x_{i,t+1} - x_{i,t+1}^u|. \quad (18)$$

Therefore, the portfolio turnover at period $t + 1$ can be written as:

$$Tur_{t+1} = \sum_{i=1}^N |x_{i,t+1} - x_{i,t+1}^u|. \quad (19)$$

In this paper, the average turnover over the holding period is generally reported. It corresponds to the average of the expressions given by (19) over all t .

2.6. Percentage loss

This loss corresponds to the difference between the portfolio return without taking into account TC and the portfolio return taking into account TC. Let W_T denote the total value of the portfolio at the very last period, which corresponds to the liquidation period. Let denote W_0 the value of the portfolio at the very first period, which corresponds to the initial investment. The total return of the portfolio without taking into account TC, $R^{without TC}$, can be expressed as follows:

$$R^{without TC} = \frac{W_T - W_0}{W_0}. \quad (20)$$

Adding TC, either per share, per percentage, or per trade, we obtain the following total portfolio return $R_j^{with TC}$:

$$R_j^{with TC} = \frac{W_T - W_0 - TC_j}{W_0}, \quad (21)$$

where TC_j denotes a specific type of TC, either per share, per percentage, or per trade and

$$TC_j = \sum_{t=0}^T \sum_{i=1}^N TC_{j,i,t} \quad (22)$$

where j includes both entry and exit costs. Therefore, the loss L_j associated with a specific type of TC j can be simply expressed by the following equation:

$$L_j = R^{without TC} - R_j^{with TC}. \quad (23)$$

For the purpose of our study, the value of L_j has been annualized in order to observe how much is lost per year. It will be used to assess portfolio performance.

3 Analysis

TC can have a different impact on portfolio performance depending on the composition of the portfolio, the holding period, and the adopted trading strategy. Next we will analyze the interaction of these factors with the types of TC described in Section 2. The analysis is mainly conducted from the perspective of a retail investor trading in the Peruvian market. Nevertheless, U.S. stocks and transaction fees are included to enhance the scope of our analysis.

3.1. Stock selection procedure

To analyze the impact of TC from the perspective of an investor trading on the Peruvian stock market, we have created four portfolios of fifteen (15) stocks each. Two of them are composed of Peruvian stocks only, while the two others consist of comparable American portfolios and contain only stocks listed on the NYSE.

The first portfolio is composed of fifteen Peruvian stocks with the highest market capitalization on the BVL on the date of December 31st 2016. Let Peruvian Blue Chips (PBC) denote this first portfolio. The second portfolio includes fifteen Peruvian stocks with the lowest market capitalization on the BVL on the date of December 31st 2016 and it will be denoted as Peruvian Pink Sheets (PPS). For our comparisons with the American market, two other portfolios were created by picking similar stocks in terms of market capitalization, price, and industry, on the NYSE, and on the date of December 31st 2016. Therefore, the third portfolio consists of fifteen U.S. stocks comparable to the ones of the PBC portfolio and it will be denoted as American Blue Equivalents (ABE). The fourth portfolio is composed of fifteen U.S. stocks equivalent to the PPS portfolio. Let American Pink Equivalents (APE) denote this fourth portfolio. Table 3.1

offers an insight of the composition of each portfolio.⁷ Besides, all stock prices in Peruvian Nuevo Sol (PEN) have been converted to U.S. Dollars (USD) and it has been assumed that we can trade any amount at the quoted prices.

3.2. Initial amount invested

In order to analyze the relationship between initial investment and TC, the experiment has been run with thirteen different amounts of USD for each portfolio: \$10,000, \$20,000, \$30,000, \$60,000, \$100,000, \$200,000, \$300,000, \$600,000, \$1,000,000, \$2,000,000, \$3,000,000, \$6,000,000, and \$10,000,000.

3.3. Holding period

Each portfolio has been analyzed for three different time frames. Our data for stock prices and market capitalizations has been extracted successively for nine (9), five (5), and three (3) years. All data has been extracted on a monthly basis. The nine-year period of analysis goes from January, 31st 2008 to December, 31st 2016. The five-year period of analysis goes from January, 31st 2008 to December, 31st 2012. The three-year period of analysis goes from January, 31st 2008 to December, 31st 2010. The beginning period of January, 31st 2008 has been chosen in order to cover the Financial Crisis and its effects on stock prices. Indeed, the Dow-Jones fell over half from a high of 14,165 on October 9th, 2007 to a low of 6,926 on March 5th, 2009 (Federal Reserve Bank of Atlanta, 2009). We wanted to analyze if similar effects on stock prices are observable on the BVL and their possible impact on TC.

⁷ The data for each stock consists of the end of the day last prices and current market capitalizations over a monthly time period that started on January 31st 2008 and ended on December 31st 2016. It has been extracted through Bloomberg.

Table 3.1. Compositions of portfolios PBC, PPS, ABE, and APE

	Ticker Name	Short Name	Industry
PBC	CREDITC1 PE Equity	BANCO DE CREDI-C	Banking
	CONTINC1 PE Equity	BBVA BANCO CONTI	Banking
	SCOTIAC1 PE Equity	SCOTIABANK PER-C	Banking
	ENGEPEC1 PE Equity	ENEL GENERACION	Energy
	ENGIEC1 PE Equity	ENGIE ENERGIA PE	Energy
	BACKUSI1 PE Equity	UNION CERV BAC-I	Food & Beverages
	ALICORC1 PE Equity	ALICORP-C	Food & Beverages
	BUENAVC1 PE Equity	BUENAVENTURA-COM	Mining
	CVERDEC1 PE Equity	SOCIEDAD MINERA	Mining
	LUSURC1 PE Equity	LUZ SUR-COMUN	Mining
	TELEFBC1 PE Equity	TELEF PERU-B	Mining
	MILPOC1 PE Equity	COMPANIA MINERA	Mining
	MINSURI1 PE Equity	MINSUR-INV	Mining
VOLCABC1 PE Equity	VOLCAN CIA MIN-B	Mining	
UNACEMC1 PE Equity	UNACEM SAA	Mining	
PPS	POMALCC1 PE Equity	EMP Agroindustrial Pomalca-C	Agroindustrial
	SNJACIC1 PE Equity	Agroindust San Jacinto-Comm	Agroindustrial
	CASAGRC1 PE Equity	Casa Grande SAA	Agroindustrial
	TUMANC1 PE Equity	EMP Agroindustrial Tuman-Cmn	Agroindustrial
	LAREDOC1 PE Equity	Agroindustrial Laredo-Cm	Agroindustrial
	CARTAVC1 PE Equity	Cartavio SAA	Agroindustrial
	CAUCHOI1 PE Equity	Lima Caucho SAI	Auto Parts
	RAURAI1 PE Equity	Compania Minera Raura SA-Inv	Base Metals
	MOROCO11 PE Equity	San Ignacio De Morococha-T	Base Metals
	MINCORI1 PE Equity	Soc Minera Corona SA-Inv	Base Metals
	HIDRA2C1 PE Equity	Hidrandina SA-A2 Shares	Energy
	AUSTRAC1 PE Equity	Austral Group SAA	Food & Beverages
	INVCENC1 PE Equity	Inversiones Centenario-Comun	Real Estate
RELAPAC1 PE Equity	Refineria La Pampilla SAA	Refining	
SIDERC1 PE Equity	Empresa Siderurgica Peru SAA	Steel Producer	
ABE	CMA US Equity	Comerica Inc	Banking
	SNV US Equity	Synovus Financial Corp	Banking
	TFSL US Equity	TFS Financial Corp	Banking
	DRQ US Equity	Dril-Quip Inc	Energy
	CRZO US Equity	Carrizo Oil & Gas Inc	Energy
	NSH US Equity	Nustar GP Holdings LLC	Energy
	HLX US Equity	Helix Energy Solutions Group	Energy
	CNX US Equity	Consol Energy Inc	Mining
	RGLD US Equity	Royal Gold Inc	Mining
	RRC US Equity	Range Resources Corp	Mining
	SWC US Equity	Stillwater Mining Co	Mining
	ARLP US Equity	Alliance Resource Partners	Mining
	CDE US Equity	Coeur Mining Inc	Mining
SHOO US Equity	Steven Madden Ltd	Retail	
HLF US Equity	Herbalife Ltd	Retail	
APE	CRGS US Equity	Curaegis Technologies Inc	Auto Parts
	GMO US Equity	General Moly Inc	Base Metals
	UAMY US Equity	United States Antimony Corp	Base Metals
	SNAK US Equity	Inventure Foods Inc	Food & Beverages
	BRID US Equity	Bridgford Foods Corp	Food & Beverages
	AMNF US Equity	Armanino Foods of Distinct	Food & Beverages
	RMCF US Equity	Rocky Mountain Choc Fact Inc	Food & Beverages
	FAC US Equity	First Acceptance Corp	Insurance
	LODE US Equity	Comstock Mining Inc	Mining
	ECPN US Equity	El Capitan Precious Metals	Mining
	XPL US Equity	Solitario Exploration & Roy	Mining
	HNRG US Equity	Hallador Energy Co	Mining
	LFVN US Equity	Lifeadvantage Corp	Specialty Pharma
NAII US Equity	Natural Alternatives Intl	Specialty Pharma	
CYAN US Equity	Cyanotech Corp	Specialty Pharma	

3.4. Parameters for TC

Each TC has parameters that determines how expensive it is. The TC per share has a trading fee in dollar θ_i that is applied for each single stock bought or sold on the stock market. The value of θ_i used is \$0.01 per share and refers to the common trading fee used by American online brokerage firms such as Lightspeed Trading or Trade Station Securities.⁸ In addition, those firms apply a minimum trading cost in dollars, α_i , of \$1 per trade. Finally, the SEC applies a regulatory fee ρ_i on all stock sales values only. The value of ρ_i used is 0.0000238.⁹

Concerning the TC per percentage, the fixed percentage per stock value β_i is determined by the broker making the transaction. The value of β_i used is 0.55% and refers to the average trading fee applied by stockbrokers in Peru. As for the TC per share, the TC per percentage faces a minimum trading cost in dollar γ_i . The value of γ_i used is \$25 and corresponds to an average of Peruvian brokers. Besides, the TC per percentage faces a regulatory fee μ_i applied by the BVL and The value of μ_i used is 0.08295%.¹⁰ Last but not least, the TC per trade faces a trading cost ε_i of \$7.97. This cost is an average of the trading cost used by American online brokerage firms such as E-Trade, Charles Schwab, TD Ameritrade, or Tradeking¹¹. As for TC per share, TC per trade faces a regulatory fee ρ_i .

3.5. Influence of the portfolio composition

⁸ Parameters taken at the date of March 1st, 2017 on the websites lightspeed.com and tradestation.com

⁹ Value picked from the Securities and Exchange Commission website: www.sec.gov/news/pressrelease/

¹⁰ Value including *retribución BVL*, *fondo de garantía*, *retribuciones Cavali*, and *contribución SMV*.

¹¹ Parameters taken at the date of March 1st, 2017 on the websites etrade.com, schwab.com, tdameritrade.com, and tradeking.com

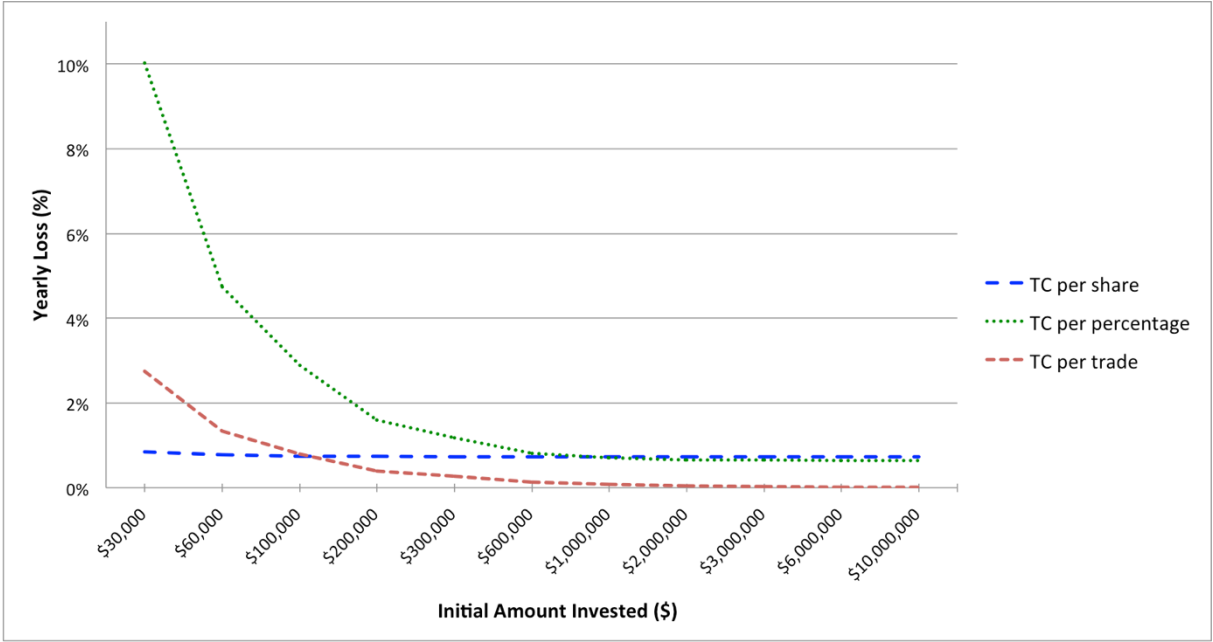
Each of the portfolios described in Section 3.1 has been analyzed over a period of five years and under an EW strategy given by (1). Under the perspective of an investor trading on the Peruvian market, the PBC portfolio has first been analyzed. This section aims at determining the impact of different initial amounts invested on the behavior of the different TC structures.

Figure 1 shows the yearly loss per initial amount invested for each type of TC for the PBC portfolio. First, we note that TC per share are efficient for small initial amounts invested in the PBC portfolio. The yearly loss is always lower than 1% for any amount invested equal or above \$20,000. TC per share also tend to be the least volatile among all types of TC. Second, both TC per percentage and TC per trade become better than TC per share once a certain initial amount invested is reached. TC per percentage have a yearly loss that becomes lower than the one for TC per share for any amount invested equal or above \$1,000,000. In the same connection, TC per trade are more efficient than TC per share for any amount invested equal or above \$200,000. Third, if we compare both TC per percentage and TC per trade, TC per trade are always a better choice since they face lower yearly losses for any initial amount invested.

Table 3.2 presents the yearly loss per initial amount invested for each type of TC and portfolio. Keeping the same strategy and holding period, similar conclusions can be drawn for both the PPS portfolio and the APE portfolio. As a matter of fact, TC per share remain the cheapest kind for small amounts invested for both the PPS and APE portfolios. However, once a specific level of initial amount invested is reached, TC per percentage and TC per trade are preferable. TC per trade are always better than TC per percentage. Results are a little bit different for the ABE portfolio. Indeed, TC per percentage never become preferable to TC per share. Also, TC per trade become better than TC per share only when a large initial investment is made: at least

\$2,000,000 has to be invested. The reason beyond this result is that the ABE portfolio includes stocks with higher prices and lower volatilities.

Figure 1. Yearly loss per initial amount invested for the PBC portfolio over a 5-year period and under an EW strategy



Besides, portfolios composed of stocks with large market capitalizations (PBC and ABE) tend to have less yearly loss than those with small market capitalizations (PPS and APE). This increase in yearly loss for both the PPS portfolio and the APE portfolio can be explained by lower stock prices and a higher volatility compared to the PBC and the ABE portfolios.

Entry and exit costs do not affect the portfolio returns that much. Table 3.3 presents the difference in yearly loss between TC including entry and exit costs and TC excluding entry and exit costs. The average loss without entry and exit costs is lower by 0.01% to 0.8% than when including entry and exit costs. The trends observed for the four portfolios are the same as when entry and exit costs are not taken into account.

Table 3.2. Yearly loss per initial amount invested for each type of portfolio over a 5-year period and under an EW strategy

	Amount	Yearly loss - TC per share	Yearly loss - TC per percentage	Yearly loss - TC per trade
PBC	\$ 30 000	0,85%	10,02%	2,75%
	\$ 100 000	0,75%	2,89%	0,80%
	\$ 300 000	0,73%	1,18%	0,26%
	\$ 1 000 000	0,73%	0,71%	0,08%
	\$ 3 000 000	0,73%	0,65%	0,03%
	\$ 10 000 000	0,73%	0,64%	0,01%
PPS	\$ 30 000	2,43%	20,14%	4,72%
	\$ 100 000	2,26%	4,74%	1,33%
	\$ 300 000	2,24%	1,77%	0,44%
	\$ 1 000 000	2,24%	0,95%	0,13%
	\$ 3 000 000	2,24%	0,84%	0,04%
	\$ 10 000 000	2,24%	0,82%	0,01%
ABE	\$ 30 000	0,46%	15,00%	3,83%
	\$ 100 000	0,15%	3,92%	1,09%
	\$ 300 000	0,08%	1,53%	0,36%
	\$ 1 000 000	0,06%	0,91%	0,11%
	\$ 3 000 000	0,06%	0,84%	0,04%
	\$ 10 000 000	0,06%	0,83%	0,01%
APE	\$ 30 000	1,80%	6,73%	1,88%
	\$ 100 000	1,74%	2,26%	0,55%
	\$ 300 000	1,73%	1,28%	0,18%
	\$ 1 000 000	1,73%	1,08%	0,06%
	\$ 3 000 000	1,73%	1,06%	0,02%
	\$ 10 000 000	1,73%	1,06%	0,01%

Table 3.3. Difference in yearly loss between TC including entry and exit costs and TC excluding entry and exit costs, for each portfolio, under an EW strategy, over a 5-year period, and for the PBC portfolio

	Amount	Difference in yearly loss TC per share	Difference in yearly loss TC per percentage	Difference in yearly loss TC per trade
PBC	\$ 30 000	0,27%	0,54%	0,08%
	\$ 100 000	0,27%	0,33%	0,02%
	\$ 300 000	0,27%	0,31%	0,01%
	\$ 1 000 000	0,27%	0,31%	0,00%
	\$ 3 000 000	0,27%	0,31%	0,00%
	\$ 10 000 000	0,27%	0,31%	0,00%
PPS	\$ 30 000	0,78%	1,07%	0,14%
	\$ 100 000	0,77%	0,38%	0,04%
	\$ 300 000	0,77%	0,34%	0,01%
	\$ 1 000 000	0,77%	0,33%	0,00%
	\$ 3 000 000	0,77%	0,33%	0,00%
	\$ 10 000 000	0,77%	0,33%	0,00%
ABE	\$ 30 000	0,02%	0,74%	0,11%
	\$ 100 000	0,02%	0,36%	0,03%
	\$ 300 000	0,02%	0,33%	0,01%
	\$ 1 000 000	0,02%	0,32%	0,00%
	\$ 3 000 000	0,02%	0,32%	0,00%
	\$ 10 000 000	0,02%	0,32%	0,00%
APE	\$ 30 000	0,28%	0,44%	0,05%
	\$ 100 000	0,28%	0,32%	0,02%
	\$ 300 000	0,28%	0,31%	0,01%
	\$ 1 000 000	0,28%	0,31%	0,00%
	\$ 3 000 000	0,28%	0,31%	0,00%
	\$ 10 000 000	0,28%	0,31%	0,00%

3.6. Influence of the holding period

The PBC portfolio has been analyzed changing the holding period to three and nine years. Table 3.4 depicts the yearly loss associated with each holding period for the PBC portfolio. For three and nine years, we obtain similar results to the PBC portfolio analyzed under a five-year period. First, TC per share remain the most efficient type of TC when dealing with small amounts. However, once a certain level is reached, it is better to switch to TC per percentage (between \$600,000 and \$1,000,000) or to TC per trade (between \$100,000 and \$200,000). Once again, TC per trade are preferable to TC per percentage since the yearly loss for TC per trade is always lower. Therefore, the holding period does not influence the performance of the different types of TC for the PBC portfolio.

Table 3.4. Yearly loss per initial amount invested for each holding period for the PBC portfolio over a 5-year period and under an EW strategy

	Amount	Yearly loss - TC per share	Yearly loss - TC per percentage	Yearly loss - TC per trade
3 years	\$ 30 000	1,20%	10,64%	3,09%
	\$ 100 000	1,08%	3,40%	0,91%
	\$ 300 000	1,06%	1,53%	0,30%
	\$ 1 000 000	1,06%	1,00%	0,09%
	\$ 3 000 000	1,06%	0,92%	0,03%
	\$ 10 000 000	1,06%	0,91%	0,01%
5 years	\$ 30 000	0,85%	10,02%	2,75%
	\$ 100 000	0,75%	2,89%	0,80%
	\$ 300 000	0,73%	1,18%	0,26%
	\$ 1 000 000	0,73%	0,71%	0,08%
	\$ 3 000 000	0,73%	0,65%	0,03%
	\$ 10 000 000	0,73%	0,64%	0,01%
9 years	\$ 30 000	1,05%	16,30%	3,26%
	\$ 100 000	0,92%	3,26%	0,90%
	\$ 300 000	0,90%	1,17%	0,29%
	\$ 1 000 000	0,90%	0,64%	0,09%
	\$ 3 000 000	0,90%	0,57%	0,03%
	\$ 10 000 000	0,90%	0,56%	0,01%

Also, the holding period helps dilute the large entry and exit costs: the larger the holding period, the more diluted these costs. Table 3.5 shows the difference in yearly loss between TC including entry and exit costs and TC excluding entry and exit costs, per holding period and for the PBC

portfolio and Table 3.6 illustrates the decline of average turnovers over time. The influence of both the entry and exit costs on the portfolio performance decreases when the holding period increases. This could be explained by the fact that the ratio entry-exit cost to total TC decreases over time. Indeed, these entry and exit costs remain pretty much the same for these three time frames: the entry costs are in fact the same for the three years, and the exit costs vary just a little bit due to price fluctuations at the very last period. On the other hand, the total TC increases when the holding period increases, simply because there are more periods and in time more transactions occur.

Table 3.5. Difference in yearly loss between TC including entry and exit costs and TC excluding entry and exit costs, per holding period, under an EW strategy, and for the PBC portfolio

	Amount	Difference in yearly loss TC per share	Difference in yearly loss TC per percentage	Difference in yearly loss TC per trade
3 years	\$ 30 000	0,49%	0,83%	0,14%
	\$ 100 000	0,49%	0,58%	0,04%
	\$ 300 000	0,49%	0,57%	0,01%
	\$ 1 000 000	0,49%	0,56%	0,00%
	\$ 3 000 000	0,49%	0,56%	0,00%
	\$ 10 000 000	0,49%	0,56%	0,00%
5 years	\$ 30 000	0,27%	0,54%	0,08%
	\$ 100 000	0,27%	0,33%	0,02%
	\$ 300 000	0,27%	0,31%	0,01%
	\$ 1 000 000	0,27%	0,31%	0,00%
	\$ 3 000 000	0,27%	0,31%	0,00%
	\$ 10 000 000	0,27%	0,31%	0,00%
9 years	\$ 30 000	0,21%	0,76%	0,06%
	\$ 100 000	0,20%	0,20%	0,01%
	\$ 300 000	0,20%	0,17%	0,00%
	\$ 1 000 000	0,20%	0,17%	0,00%
	\$ 3 000 000	0,20%	0,17%	0,00%
	\$ 10 000 000	0,20%	0,17%	0,00%

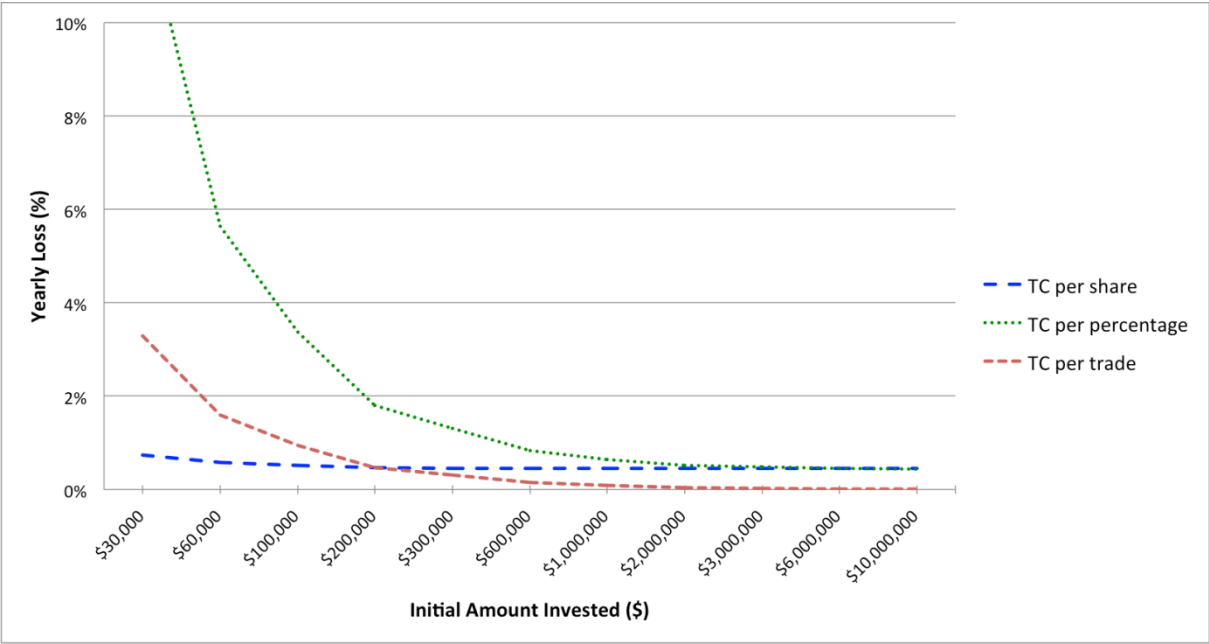
Table 3.6. Average turnovers per holding period for each type of portfolio

Portfolio	Turnover		
	3 years	5 years	9 years
PBC	6,84%	6,02%	5,93%
PPS	8,72%	7,59%	7,48%
ABE	9,70%	8,51%	8,12%
APE	14,53%	13,16%	12,12%

3.7. Influence of the trading strategy

The three trading strategies of Section 2.1 have been analyzed. The PBC portfolio has been examined under a MC strategy¹² over a five-year period to compare results found with the EW strategy. Figure 2 plots the yearly loss per initial amount invested for each type of TC for the PBC portfolio under a MC strategy given by (2). The MC strategy faces the same dynamics as the EW strategy, in the case of the PBC portfolio. Indeed, TC per share are efficient for small amounts invested. Also, TC per trade and TC per percentage overcome TC per share once a certain minimum initial investment is reached (\$10,000,000 and \$300,000 respectively). Comparing Figure 1 and Figure 2, we observe that TC per percentage and TC per trade become less efficient under a MC strategy. Indeed, switching to one of those two kinds of TC is done for larger initial amounts invested than under a EW strategy. Finally, TC per trade are always better than TC per percentage.

Figure 2. Yearly loss per initial amount invested for the PBC portfolio over a 5-year period and under a MC strategy



¹² All the data for market capitalizations for all stocks have been extracted through Bloomberg, within the same time frames as the extraction of the stocks' last prices described at the footnote ⁴.

Concerning TC per share, those are less expensive under the MC strategy: the yearly loss associated is smaller than under an EW strategy and this is the case for all portfolios. Table 3.7 presents the yearly loss per initial amount invested for all portfolios. These lower TC incurred under the MC strategy can be explained by a decrease in turnover. Table 3.8 shows that turnovers using a MC strategy are much smaller than when using an EW strategy, and for any type of portfolio. Concerning TC per trade, those are slightly more expensive when using the MC strategy: the yearly loss associated is higher than under an EW strategy. This is the case for the PBC portfolio as well as the others. Finally, TC per percentage under a MC strategy become more attractive than an EW strategy when the initial amount invested gets bigger, i.e. \$1,000,000. This observation is explained by the fact that bigger initial investments imply bigger trading costs when rebalancing the portfolio, which eventually neglect the high minimum trading cost of the TC per percentage.

Table 3.7. Yearly loss per initial amount invested for each type of portfolio over a 5-year period and under a MC strategy

	Amount	Yearly loss - TC per share	Yearly loss - TC per percentage	Yearly loss - TC per trade
PBC	\$ 30 000	0,75%	12,40%	3,30%
	\$ 100 000	0,51%	3,38%	0,95%
	\$ 300 000	0,46%	1,31%	0,31%
	\$ 1 000 000	0,45%	0,65%	0,09%
	\$ 3 000 000	0,45%	0,48%	0,03%
	\$ 10 000 000	0,45%	0,44%	0,01%
PPS	\$ 30 000	1,73%	30,79%	5,92%
	\$ 100 000	1,34%	5,85%	1,63%
	\$ 300 000	1,25%	2,08%	0,53%
	\$ 1 000 000	1,23%	0,93%	0,16%
	\$ 3 000 000	1,23%	0,64%	0,05%
	\$ 10 000 000	1,23%	0,55%	0,02%
ABE	\$ 30 000	0,72%	38,62%	6,41%
	\$ 100 000	0,22%	6,28%	1,75%
	\$ 300 000	0,08%	2,15%	0,57%
	\$ 1 000 000	0,03%	0,88%	0,17%
	\$ 3 000 000	0,02%	0,54%	0,06%
	\$ 10 000 000	0,02%	0,42%	0,02%
APE	\$ 30 000	1,01%	25,00%	7,85%
	\$ 100 000	0,44%	7,66%	2,08%
	\$ 300 000	0,29%	2,55%	0,67%
	\$ 1 000 000	0,24%	1,03%	0,20%
	\$ 3 000 000	0,23%	0,63%	0,07%
	\$ 10 000 000	0,23%	0,51%	0,02%

Table 3.8. Average turnovers per trading strategy for each type of portfolio

Portfolio	Turnover	
	EW	MC
PBC	6,02%	2,42%
PPS	7,59%	3,14%
ABE	8,51%	0,65%
APE	13,16%	1,36%

The third strategy analyzed is the MZ strategy. Due to a lack of data for Peruvian stocks, a new portfolio composed of twelve (12) ABE stocks has been built to analyze the effects of this strategy on the different kinds of TC. Let New ABE denote this new portfolio created. Appendix 1 presents the stock composition of this New ABE. To do so, the data from five years before the period of analysis was required to compute the mean vector and covariance matrix. For instance, the weights of the first period as on January, 31st 2008, have been determined using estimates from the monthly data of the past five years, as between January, 31st 2003 and December, 31st 2007. Then, the weights of the second period have been calculated using “rolling windows”, that is to say shifting the past data forward from one period, and so on and so forth for the following periods.

Table 3.9 shows the yearly loss per TC for each trading strategy. First, TC per share remain the most efficient type of TC for any initial amounts invested under \$2,000,000. Once this level is reached, TC per trade become preferable. Second, TC per percentage are inefficient when investing in the New ABE portfolio: the yearly loss associated with TC per percentage is always higher than TC per share and TC per trade.

Since a MZ strategy is characterized by active portfolio management decisions, the average monthly turnover reaches 17.47%, which is much more than when investing under an EW or a MC strategy. Indeed, the average monthly turnover associated with an EW strategy is 8.75%, and the average monthly turnover associated with a MC strategy is 0,69%. The large turnovers associated with the MZ strategy is explained by the fact that maximizing the Sharpe Ratio reallocates very different weights for the stocks from period to period. However, the reason why the MZ strategy is not the most expensive one might be because the rebalancing incurs stocks with large prices, which at the end results in lower TC.

TC per share under a MZ strategy are higher than under an EW strategy. Also, TC per share under an EW strategy are higher than under a MC strategy. This is the case for any initial amount invested above \$300,000. The reason beyond this result is that higher average turnovers are observed when active trading strategies are adopted, for any initial amount invested above \$300,000. As a matter of fact, the MZ strategy faces the highest turnover and the highest TC.

Table 3.9. Yearly loss per initial amount invested for each type of trading strategy over a 5-year period and for the New ABE portfolio

	Amount	Yearly loss - TC per share	Yearly loss - TC per percentage	Yearly loss - TC per trade
MZ	\$ 30 000	0,17%	14,49%	3,60%
	\$ 100 000	0,08%	4,21%	1,03%
	\$ 300 000	0,07%	2,26%	0,34%
	\$ 1 000 000	0,07%	1,70%	0,10%
	\$ 3 000 000	0,07%	1,56%	0,04%
	\$ 10 000 000	0,07%	1,50%	0,01%
MC	\$ 30 000	0,60%	25,70%	5,31%
	\$ 100 000	0,19%	5,27%	1,47%
	\$ 300 000	0,07%	1,88%	0,48%
	\$ 1 000 000	0,03%	0,81%	0,14%
	\$ 3 000 000	0,02%	0,52%	0,05%
	\$ 10 000 000	0,02%	0,43%	0,01%
EW	\$ 30 000	0,37%	11,40%	3,05%
	\$ 100 000	0,13%	3,20%	0,88%
	\$ 300 000	0,07%	1,33%	0,29%
	\$ 1 000 000	0,06%	0,88%	0,09%
	\$ 3 000 000	0,06%	0,83%	0,03%
	\$ 10 000 000	0,06%	0,83%	0,01%

The EW strategy faces the second highest turnover and the second highest TC. The MC strategy faces the third highest turnover and the third highest TC. Besides, the fixed component for TC per share and TC per percentage increases a lot the cost of trading. In this connection, higher TC are associated with higher average turnovers. Therefore, there is a positive correlation between TC per share and average turnover: for large amounts invested (i.e. \$300,000 or higher), the higher the average turnover, the higher the TC per share.

Moreover, TC per percentage under a MZ strategy are higher than under an EW strategy. TC per percentage under an EW strategy are higher than under a MC strategy. This is the case for any initial amount invested above \$1,000,000. This result is also due to higher average turnovers the portfolio faces when investing under a MZ or an EW strategy than under a MC strategy, as well as large stock inflows and outflows. Finally, it is not possible to conclude on TC per trade: active portfolio management strategies do not seem to affect the performance of TC per trade.

3.8. Influence of the number of stocks

Four portfolios, each of them containing sixty (60), forty-five (45), thirty (30), and fifteen (15) stocks have been created in order to assess the impact of the number of stocks on TC. These portfolios have been built selecting random stocks from our four previous portfolios. The portfolio of 60 stocks contains all stocks analyzed. The portfolio of forty-five stocks contains forty-five of the sixty stocks. The portfolio of thirty stocks contains thirty of the sixty stocks. The portfolio of fifteen stocks contains fifteen of the sixty stocks. Each portfolio has been analyzed under an EW strategy, over a five-year period. Appendices 2 to 5 present the composition of those three new portfolios. Table 3.10 shows the yearly loss per initial amount

invested for those three portfolios and Table 3.11 presents the average turnovers per number of stocks within the portfolio.

Table 3.10. Yearly loss per initial amount invested for different number of stocks over a 5-year period and under an EW strategy

	Amount	Yearly loss - TC per share	Yearly loss - TC per percentage	Yearly loss - TC per trade
15 stocks	\$ 30 000	2,80%	7,94%	2,21%
	\$ 100 000	2,71%	2,51%	0,65%
	\$ 300 000	2,69%	1,28%	0,22%
	\$ 1 000 000	2,68%	1,01%	0,07%
	\$ 3 000 000	2,68%	0,98%	0,02%
	\$ 10 000 000	2,68%	0,97%	0,01%
30 stocks	\$ 30 000	2,02%	21,18%	5,11%
	\$ 100 000	1,74%	5,15%	1,44%
	\$ 300 000	1,68%	1,95%	0,47%
	\$ 1 000 000	1,67%	1,08%	0,14%
	\$ 3 000 000	1,67%	0,95%	0,05%
	\$ 10 000 000	1,67%	0,94%	0,02%
45 stocks	\$ 30 000	2,03%	30,00%	8,79%
	\$ 100 000	1,52%	8,53%	2,37%
	\$ 300 000	1,42%	2,85%	0,77%
	\$ 1 000 000	1,40%	1,21%	0,23%
	\$ 3 000 000	1,39%	0,91%	0,08%
	\$ 10 000 000	1,39%	0,87%	0,02%
60 stocks	\$ 30 000	2,31%	45,00%	14,11%
	\$ 100 000	1,51%	13,37%	3,52%
	\$ 300 000	1,34%	4,03%	1,13%
	\$ 1 000 000	1,30%	1,49%	0,33%
	\$ 3 000 000	1,29%	0,95%	0,11%
	\$ 10 000 000	1,29%	0,86%	0,03%

Table 3.11. Average turnovers per number of stocks in the portfolio

Strategy	Turnover			
	15 stocks	30 stocks	45 stocks	60 stocks
EW	11,38%	10,68%	9,72%	9,40%

A few observations can be made depending on the number of stocks within the portfolio. First, TC per share decrease when the number of shares in the portfolio increases, for any initial amount invested above or equal to \$100,000. This is due to a decrease in rebalancing when the number of stocks increases. TC per share remain the most attractive kind of TC for small

amounts invested compared to the other types of TC. However, once a specific initial amount invested is reached, switching to either TC per percentage or TC per trade is preferable to TC per share. Second, TC per percentage also decrease when the number of shares in the portfolio increases, for any large initial amount greater than \$6,000,000. Third, TC per trade increase when the number of shares in the portfolio increases. The previous results can be explained by the fact that having more stocks in a portfolio implies less average turnovers per stock under an EW strategy. Indeed, average turnovers tend to decrease when the number of stocks increases. There are actually less weight variations for each stock when the portfolio becomes bigger. Finally, TC per percentage never become better than TC per trade when the number of stocks in the portfolio increases. TC per trade are always cheaper than TC per percentage, for all three kinds of portfolios (i.e. 15, 30, 45, and 60 stocks) and are preferable to any other kind of TC for any minimum initial amount invested greater than \$300,000.

4 Making the TC per percentage competitive

It has been concluded in Section 3 that TC per percentage is the most expensive TC structure. Its fixed component β_i is actually very high: an investor trading on the BVL and using a broker has to pay a minimum trading cost of \$25. In this section, we explain the last experiment we run with the objective to make TC per percentage competitive and see how the parameters described in Section 3.4 should change in order for an investor to be able to actively trade on the BVL. The experiment has been run for the PBC portfolio, under an EW strategy, and over a five-year period.

To make TC per percentage competitive, a minimum fixed TC β_i of \$1 has been imposed. Indeed, TC per share also suggest a minimum fixed TC α_i of \$1, and that is why the value of \$1 for β_i has been chosen. Figure 4 shows that the yearly loss associated with TC per percentage

is always smaller than the one for TC per share. TC per percentage becomes the cheapest type of TC structure for small amounts invested. Besides, all yearly losses associated with TC per percentage are below 0.8%, for any initial amount invested. This means that it becomes more attractive for an investor to actively trade on the BVL when β_i is equal to \$1 than when β_i is equal to \$25. Table 5.1 presents the new yearly losses associated per type of TC for the PBC portfolio. Also, TC per percentage become constant and quickly reach a limit when the initial amount invested increases. The yearly loss associated with TC per percentage tends to the value 0.64%.

Therefore, in order to successfully invest on the BVL, the β_i coefficient has to be lowered. A β_i coefficient equal to \$25 represents too much TC especially for highly volatile stocks. With a lower β_i , investing on the BVL becomes more competitive and close to trading on the NYSE with a TC per share structure for the same types of stock.

Figure 3. Yearly loss per initial amount invested for the PBC portfolio under an EW strategy and over a 5-year period, considering the change in the structure of TC per percentage

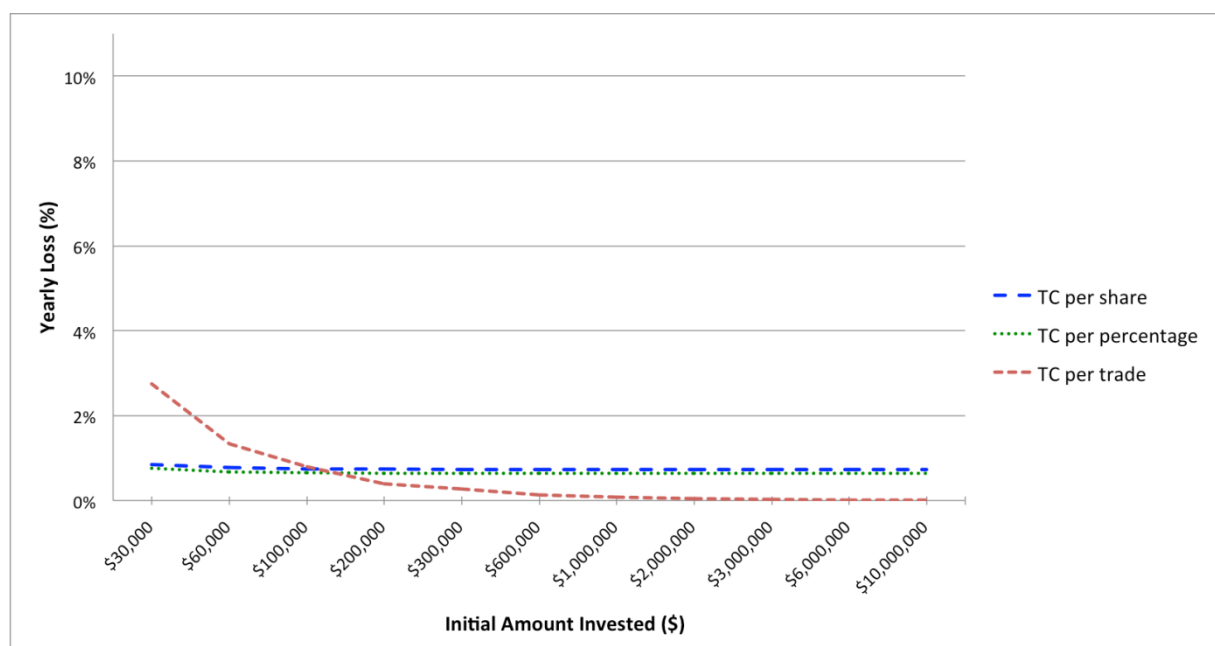


Table 5.1. Yearly loss per initial amount invested for the PBC portfolio under an EW strategy and over a 5-year period, considering the change in the structure of TC per percentage

	Amount	Yearly loss - TC per share	Yearly loss - TC per percentage	Yearly loss - TC per trade
PBC	\$ 30 000	0,85%	0,76%	2,75%
	\$ 100 000	0,75%	0,65%	0,80%
	\$ 300 000	0,73%	0,64%	0,26%
	\$ 1 000 000	0,73%	0,64%	0,08%
	\$ 3 000 000	0,73%	0,64%	0,03%
	\$ 10 000 000	0,73%	0,64%	0,01%

5 Conclusions and recommendations

The impact of different types of TC (per share, per percentage, and per trade) has been analyzed while focusing on different dimensions: type and number of stocks in the portfolios, holding periods, and trading strategies.

Using average parameters for the types of TC considered, trends have been observed and are independent from those dimensions. First, TC per share are the preferable type of TC when investing a relatively small initial amount. Second, once a specific level of initial amount is reached, investor should switch to either TC per percentage or TC per trade, whichever become cheaper than TC per share. Third, TC per trade are always better than TC per percentage for all dimensions considered.

Moreover, each dimension gives an insight on the cost of trading. First, the effect of the portfolio composition analyzed in Section 3.5 shows that investing in small market capitalization stocks is relatively more expensive than investing in large market capitalization stocks. Second, the effect of the holding period analyzed in Section 3.6 shows that entry and exit costs are diluted when an investor carries a portfolio for longer periods. Third, the effect

of the trading strategies analyzed in Section 3.7 shows that very active portfolio management strategies such as the MZ strategy tend to have larger TC.

Our observations suggest that the Peruvian TC per percentage are currently very expensive and are efficient only if large initial amounts are invested in a portfolio. From the perspective of an investor trading on the Peruvian stock market, important losses will occur on the portfolio return due to the TC per percentage structure. As a matter of fact, if an investor decides to invest in a PBC portfolio with \$30,000, \$100,000 or \$1,000,000 in value, the cost of rebalancing will be of at least 3.6 times more than if a TC per trade structure had been adopted. In fact, the yearly loss for \$30,000, \$100,000, and \$1,000,000 under TC per percentage are 10.02%, 2.89%, and 0.71%, respectively. On the other hand, the yearly loss for \$30,000, \$100,000, and \$1,000,000 under TC per trade are 2.75%, 0.80%, and 0.08%, respectively.

A last experiment has been conducted in order to see if by changing its fixed component β_i , the TC per percentage became more attractive for the Peruvian investor. By imposing a minimum fixed TC β_i of \$1, we show that the yearly loss associated with TC per percentage is now always smaller than the one for TC per share. The yearly loss associated with TC per percentage is also smaller than TC per trade for any initial amount invested lower than \$200,000. Therefore, TC per percentage become preferable to TC per share, or TC per trade for a small initial amount invested. TC per percentage become a competitive structure and allows investors to adopt active portfolio management decisions while investing on the BVL. Finally, this modified TC policy can attract retail investors and make the BVL a less illiquid stock market.

APPENDICES

Appendix 1. New ABE portfolio composition

	Ticker Name	Short Name	Industry
New ABE	CMA US Equity	Comerica Inc	Banking
	SNV US Equity	Synovus Financial Corp	Banking
	DRQ US Equity	Dril-Quip Inc	Energy
	CRZO US Equity	Carrizo Oil & Gas Inc	Energy
	HLX US Equity	Helix Energy Solutions Group	Energy
	CNX US Equity	Consol Energy Inc	Mining
	RGLD US Equity	Royal Gold Inc	Mining
	RRC US Equity	Range Resources Corp	Mining
	SWC US Equity	Stillwater Mining Co	Mining
	ARLP US Equity	Alliance Resource Partners	Mining
	CDE US Equity	Coeur Mining Inc	Mining
	SHOO US Equity	Steven Madden Ltd	Retail

Appendix 2. Portfolio compositions for 15 stocks

	Ticker Name	Short Name	Industry
15 stocks	POMALCC1 PE Equity	EMP Agroindustrial Pomalca-C	Agroindustrial
	LAREDOC1 PE Equity	Agroindustrial Laredo-Cm	Agroindustrial
	CRGS US Equity	Curaegis Technologies Inc	Auto Parts
	TFSL US Equity	TFS Financial Corp	Banking
	MOROCO11 PE Equity	San Ignacio De Morococha-T	Base Metals
	NSH US Equity	Nustar GP Holdings LLC	Energy
	ENGEPEC1 PE Equity	ENEL GENERACION	Energy
	BACKUS11 PE Equity	UNION CERV BAC-I	Food & Beverages
	RRC US Equity	Range Resources Corp	Mining
	ARLP US Equity	Alliance Resource Partners	Mining
	ECPN US Equity	El Capitan Precious Metals	Mining
	HLF US Equity	Herbalife Ltd	Retail
	LFVN US Equity	Lifedvantage Corp	Specialty Pharma
	CYAN US Equity	Cyanotech Corp	Specialty Pharma
	SIDERC1 PE Equity	Empresa Siderurgica Peru SAA	Steel Producer

Appendix 3. Portfolio compositions for 30 stocks

	Ticker Name	Short Name	Industry
	POMALCC1 PE Equity	EMP Agroindustrial Pomalca-C	Agroindustrial
	TUMANC1 PE Equity	EMP Agroindustrial Tuman-Cmn	Agroindustrial
	LAREDOC1 PE Equity	Agroindustrial Laredo-Cm	Agroindustrial
	CARTAVC1 PE Equity	Cartavio SAA	Agroindustrial
	CRGS US Equity	Curaeigis Technologies Inc	Auto Parts
	SCOTIAC1 PE Equity	SCOTIABANK PER-C	Banking
	SNV US Equity	Synovus Financial Corp	Banking
	TFSL US Equity	TFS Financial Corp	Banking
	MOROCOII PE Equity	San Ignacio De Morococha-T	Base Metals
	UAMY US Equity	United States Antimony Corp	Base Metals
	ENGEPEC1 PE Equity	ENEL GENERACION	Energy
	DRQ US Equity	Dril-Quip Inc	Energy
	NSH US Equity	Nustar GP Holdings LLC	Energy
	BACKUSI1 PE Equity	UNION CERV BAC-I	Food & Beverages
	BRID US Equity	Bridgford Foods Corp	Food & Beverages
30 stocks	BUENAVC1 PE Equity	BUENAVENTURA-COM	Mining
	LUSURC1 PE Equity	LUZ SUR-COMUN	Mining
	MINSURI1 PE Equity	MINSUR-INV	Mining
	RRC US Equity	Range Resources Corp	Mining
	SWC US Equity	Stillwater Mining Co	Mining
	ARLP US Equity	Alliance Resource Partners	Mining
	LODE US Equity	Comstock Mining Inc	Mining
	ECPN US Equity	El Capitan Precious Metals	Mining
	AUSTRAC1 PE Equity	Austral Group SAA	Packaged Food
	SHOO US Equity	Steven Madden Ltd	Retail
	HLF US Equity	Herbalife Ltd	Retail
	LFVN US Equity	Lifedvantage Corp	Specialty Pharma
	NAII US Equity	Natural Alternatives Intl	Specialty Pharma
	CYAN US Equity	Cyanotech Corp	Specialty Pharma
	SIDERC1 PE Equity	Empresa Siderurgica Peru SAA	Steel Producer

Appendix 4. Portfolio compositions for 45 stocks

Ticker Name	Short Name	Industry
POMALCC1 PE Equity	EMP Agroindustrial Pomalca-C	Agroindustrial
CASAGRC1 PE Equity	Casa Grande SAA	Agroindustrial
TUMANC1 PE Equity	EMP Agroindustrial Tuman-Cmn	Agroindustrial
LAREDOC1 PE Equity	Agroindustrial Laredo-Cm	Agroindustrial
CARTAVC1 PE Equity	Cartavio SAA	Agroindustrial
CAUCHOI1 PE Equity	Lima Caucho SAI	Auto Parts
CRGS US Equity	Curaegis Technologies Inc	Auto Parts
CONTINC1 PE Equity	BBVA BANCO CONTI	Banking
SCOTIAC1 PE Equity	SCOTIABANK PER-C	Banking
CMA US Equity	Comerica Inc	Banking
SNV US Equity	Synovus Financial Corp	Banking
TFSL US Equity	TFS Financial Corp	Banking
MOROCO11 PE Equity	San Ignacio De Morococha-T	Base Metals
MINCOR11 PE Equity	Soc Minera Corona SA-Inv	Base Metals
UAMY US Equity	United States Antimony Corp	Base Metals
ENGEPEC1 PE Equity	ENEL GENERACION	Energy
ENGIEC1 PE Equity	ENGIE ENERGIA PE	Energy
DRQ US Equity	Dril-Quip Inc	Energy
NSH US Equity	Nustar GP Holdings LLC	Energy
HLX US Equity	Helix Energy Solutions Group	Energy
BACKUS11 PE Equity	UNION CERV BAC-I	Food & Beverages
SNAK US Equity	Inventure Foods Inc	Food & Beverages
45 stocks BRID US Equity	Bridgford Foods Corp	Food & Beverages
RMCF US Equity	Rocky Mountain Choc Fact Inc	Food & Beverages
BUENAVC1 PE Equity	BUENAVENTURA-COM	Mining
LUSURC1 PE Equity	LUZ SUR-COMUN	Mining
TELEFBC1 PE Equity	TELEF PERU-B	Mining
MINSUR11 PE Equity	MINSUR-INV	Mining
VOLCABC1 PE Equity	VOLCAN CIA MIN-B	Mining
UNACEMC1 PE Equity	UNACEM SAA	Mining
RGLD US Equity	Royal Gold Inc	Mining
RRC US Equity	Range Resources Corp	Mining
SWC US Equity	Stillwater Mining Co	Mining
ARLP US Equity	Alliance Resource Partners	Mining
LODE US Equity	Comstock Mining Inc	Mining
ECPN US Equity	El Capitan Precious Metals	Mining
XPL US Equity	Solitario Exploration & Roy	Mining
AUSTRAC1 PE Equity	Austral Group SAA	Packaged Food
INVCENC1 PE Equity	Inversiones Centenario-Comun	Real estate
RELAPAC1 PE Equity	Refineria La Pampilla SAA	Refining & mkting
SHOO US Equity	Steven Madden Ltd	Retail
HLF US Equity	Herbalife Ltd	Retail
LFVN US Equity	Lifeadvantage Corp	Specialty Pharma
NAII US Equity	Natural Alternatives Intl	Specialty Pharma
SIDERC1 PE Equity	Empresa Siderurgica Peru SAA	Steel Producer

Appendix 5. Portfolio compositions for 60 stocks

	Ticker Name	Short Name	Industry
	CARTAVC1 PE Equity	Cartavio SAA	Agroindustrial
	CASAGRC1 PE Equity	Casa Grande SAA	Agroindustrial
	LAREDOC1 PE Equity	Agroindustrial Laredo-Cm	Agroindustrial
	POMALCC1 PE Equity	EMP Agroindustrial Pomalca-C	Agroindustrial
	SNJACIC1 PE Equity	Agroindust San Jacinto-Comm	Agroindustrial
	TUMANC1 PE Equity	EMP Agroindustrial Tuman-Cmm	Agroindustrial
	CAUCHO11 PE Equity	Lima Caucho SAI	Auto Parts
	CRGS US Equity	Curaegis Technologies Inc	Auto Parts
	CMA US Equity	Comerica Inc	Banking
	CONTINC1 PE Equity	BBVA BANCO CONTI	Banking
	CREDITC1 PE Equity	BANCO DE CREDI-C	Banking
	SCOTIAC1 PE Equity	SCOTIABANK PER-C	Banking
	SNV US Equity	Synovus Financial Corp	Banking
	TFSL US Equity	TFS Financial Corp	Banking
	GMO US Equity	General Moly Inc	Base Metals
	MINCOR11 PE Equity	Soc Minera Corona SA-Inv	Base Metals
	MOROCO11 PE Equity	San Ignacio De Morococha-T	Base Metals
	RAURAI11 PE Equity	Compania Minera Raura SA-Inv	Base Metals
	UAMY US Equity	United States Antimony Corp	Base Metals
	CRZO US Equity	Carrizo Oil & Gas Inc	Energy
	DRQ US Equity	Dril-Quip Inc	Energy
	ENGEPEC1 PE Equity	ENEL GENERACION	Energy
	ENGIEC1 PE Equity	ENGIE ENERGIA PE	Energy
	HIDRA2C1 PE Equity	Hidrandina SA-A2 Shares	Energy
	HLX US Equity	Helix Energy Solutions Group	Energy
	NSH US Equity	Nustar GP Holdings LLC	Energy
	ALICORC1 PE Equity	ALICORP-C	Food & Beverages
	AMNF US Equity	Armanino Foods of Distinct	Food & Beverages
	AUSTRAC1 PE Equity	Austral Group SAA	Food & Beverages
60 stocks	BACKUS11 PE Equity	UNION CERV BAC-I	Food & Beverages
	BRID US Equity	Bridgford Foods Corp	Food & Beverages
	RMCF US Equity	Rocky Mountain Choc Fact Inc	Food & Beverages
	SNAK US Equity	Inventure Foods Inc	Food & Beverages
	FAC US Equity	First Acceptance Corp	Insurance
	ARLP US Equity	Alliance Resource Partners	Mining
BUENAVC1 PE Equity	BUENAVENTURA-COM	Mining	
CDE US Equity	Coeur Mining Inc	Mining	
CNX US Equity	Consol Energy Inc	Mining	
CVERDEC1 PE Equity	SOCIEDAD MINERA	Mining	
ECPN US Equity	El Capitan Precious Metals	Mining	
HNRG US Equity	Hallador Energy Co	Mining	
LODE US Equity	Comstock Mining Inc	Mining	
LUSURC1 PE Equity	LUZ SUR-COMUN	Mining	
MILPOC1 PE Equity	COMPANIA MINERA	Mining	
MINSUR11 PE Equity	MINSUR-INV	Mining	
RGLD US Equity	Royal Gold Inc	Mining	
RRC US Equity	Range Resources Corp	Mining	
SWC US Equity	Stillwater Mining Co	Mining	
TELEFBC1 PE Equity	TELEF PERU-B	Mining	
UNACEMC1 PE Equity	UNACEM SAA	Mining	
VOLCABC1 PE Equity	VOLCAN CIA MIN-B	Mining	
XPL US Equity	Solitario Exploration & Roy	Mining	
INVCENC1 PE Equity	Inversiones Centenario-Comun	Real estate	
RELAPAC1 PE Equity	Refineria La Pampilla SAA	Refining	
HLF US Equity	Herbalife Ltd	Retail	
SHOO US Equity	Steven Madden Ltd	Retail	
CYAN US Equity	Cyanotech Corp	Specialty Pharma	
LFVN US Equity	Lifeadvantage Corp	Specialty Pharma	
NAII US Equity	Natural Alternatives Intl	Specialty Pharma	
SIDERC1 PE Equity	Empresa Siderurgica Peru SAA	Steel Producer	

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